Lectured By Dr. H.K. Yadav
Deptt. of Mathematics Marwari College, Darbhanga
B.Sc-part-II (H), paper-III, 07-05-2020
Differential Calculus (Curvature)

Q.1. Find the intrinsic equation of the catenary

$$y = c\cosh\frac{x}{c}$$

Soln: The eqn of the catenary is

$$y = c\cosh\frac{x}{c} \quad \text{--- (I)}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \cancel{c}\sinh\frac{x}{c}\times\frac{1}{\cancel{c}}$$

$$\Rightarrow \tan\psi = \sinh\frac{x}{c} \quad \text{--- II}$$

We have, $\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\sinh^2\frac{x}{c}} = \cosh\frac{x}{c}$

or, $ds = \cosh\frac{x}{c}\,dx$

Integrating both sides, we get

$$\int ds = c\int \cosh\frac{x}{c}\,dx \Rightarrow s = c\sinh\frac{x}{c} + A \quad \text{--- III}$$

Where A is the constant of integration.

Let us suppose that at the origin, $s=0, x=0$

$\therefore$ From III, $A=0$

$\therefore s = c\sinh\frac{x}{c} \Rightarrow s = c\tan\psi$ [from 2]

which is the required intrinsic eqn of the catenary

Q.2. Find the intrinsic eqn of the evolute of the circle, viz $\theta = \frac{\sqrt{(r^2-a^2)}}{a} - \cos^{-1}\frac{a}{r}$.

taking $(a,0)$ as the fixed point.

Soln: Here, we have $\psi = \theta + \phi$ --- (I)

Now, $\theta = \frac{\sqrt{r^2-a^2}}{a} - \cos^{-1}\frac{a}{r}$ --- II

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Rest part of Q.2.

Now, $\theta = \frac{\sqrt{r^2-a^2}}{a} - \cos^{-1}\frac{a}{r}$

Diff. w.r.t. r, we get

$$\frac{d\theta}{dr} = \frac{r}{a\sqrt{r^2-a^2}} + \frac{1}{\sqrt{\left(1-\frac{a^2}{r^2}\right)}}\left(-\frac{a}{r^2}\right)$$

$$= \frac{r}{a} \times \frac{1}{\sqrt{r^2-a^2}} - \frac{a}{r} \times \frac{1}{\sqrt{r^2-a^2}}$$

$$= \frac{1}{\sqrt{r^2-a^2}} \times \frac{r^2-a^2}{r.a} = \frac{\sqrt{r^2-a^2}}{r.a}$$

$\therefore \tan\phi = r\frac{d\theta}{dr} = r \times \frac{\sqrt{r^2-a^2}}{r.a} = \frac{\sqrt{r^2-a^2}}{a}$

$\therefore \cos\phi = \frac{a}{r} \Rightarrow \phi = \cos^{-1}\frac{a}{r}$

$\therefore$ from II, $\theta = \tan\phi - \phi$

$\Rightarrow \tan\phi = \theta + \phi = \psi$

Now, $s = \int_a^r \sqrt{\left\{1 + r^2\left(\frac{d\theta}{dr}\right)^2\right\}}\, dr$

$$= \int_a^r \sqrt{\left(1 + \frac{r^2-a^2}{a^2}\right)}\, dr$$

$$= \int_a^r \frac{r}{a}\, dr = \frac{1}{2a}\left[r^2 - a^2\right]$$

$$= \frac{1}{2a}\left[a^2\tan^2\phi\right]$$

$$= \frac{a}{2}\psi^2 \quad [\because \tan\phi = \psi]$$

Proved